Contents

1 Timescales in bulk Poincare observables

Here we will explain the timescales involved in a simple, well-understood process on the Coulomb branch of the $\mathcal{N} = 4$ SYM theory, dual to a bulk process on the gravity side, and compare these timescales to the timescale for a signal to get to the boundary. This gives an example of a radially localized process on the gravity side for which we know a simple standard QFT description on timescales short compared to the time one would need to wait for a signal to be sent from the bulk event to the boundary.

Consider the $\mathcal{N}=4$ SYM theory on its Coulomb branch, with gauge symmetry breaking $U(N)\to U(N-2)\times U(2)$ below the scale of the "W boson" mass ϕ. In the Poincaré patch metric

$$ds^2=\frac{r_p^2}{R^2}(-dt_p^2+d\vec{x}^2)+\frac{R^2}{r_p^2}dr_p^2 \tag{1}$$

this corresponds to two D3-branes at position

$$r_p=\phi\alpha'. \tag{2}$$

with ϕ a canonically normalized scalar field in the $\mathcal{N}=4$ SYM theory. Here I use subscript p for Poincare coordinates.

Next, let us separate the two D-branes by a small amount, putting them at $r_{p1}=\phi_1/\alpha'$ and $r_{p2}=\phi_2/\alpha'$, with $\phi_1-\phi_2\ll\phi$. We will send them toward each other, a process which produces strings between them. In the dual QFT, this process corresponds to particle production of off-diagonal matrix elements in the $U(2)$ theory in the obvious basis.

Let us compare the timescales Δt_{prod} for this production process, Δt_{bdry} for the information about it to get to the boundary, and $\Delta t_W=m_W^{-1}=1/\phi$. (Our description in terms of symmetry breaking is only good for timescales longer than the latter.) The point is below (8)(9) but I'll first include the derivation of the timescales.

We can obtain Δt_{bdry} from the metric (1), a lightlike trajectory is

$$\frac{dr_p}{dt_p}=\frac{r_p^2}{R^2} \tag{3}$$

Integrating this, we have

$$\Delta t_{p,bdry}=\int_{r_p}^{\infty}\frac{d\tilde{r}R^2}{\tilde{r}^2}=\frac{R^2}{r_p} \tag{4}$$

In QFT terms

$$\Delta t_{p,bdry}=\frac{\sqrt{\lambda}}{\phi} \tag{5}$$

The timescale for particle production in the $U(2)$ sector is given by the relative field velocity $\dot{\phi}_{12}=\dot{\phi}_1-\dot{\phi}_2$ (up to a power of g_s which I'll suppress but is easy to include [1]):

$$\Delta t_{p,prod}\sim\dot{\phi}_{12}^{-1/2} \tag{6}$$

(Here I'm using a conservative estimate of this timescale: the statement is that this timescale is the one beyond which the physics returns to being adiabatic. There may well be shorter timescales that we could also identify.) Now $\dot{\phi}_{12}$ is limited by the bulk speed of light, $\dot{\phi}_{12}<\phi^2/\sqrt{\lambda}$. So we get

$$\Delta t_{p,prod}>\frac{\lambda^{1/4}}{\phi} \tag{7}$$

Collecting the timescales, we have

$$\Delta t_{p,W} = \frac{1}{\phi}, \quad \Delta t_{p,prod} > \frac{\lambda^{1/4}}{\phi}, \quad \Delta t_{p,bdry} = \frac{\sqrt{\lambda}}{\phi} \tag{8}$$

So for $\lambda \gg 1$, we have an interesting window

$$\Delta t_{p,W} \ll \Delta t_{p,prod} \ll \Delta t_{p,bdry} \tag{9}$$

The first inequality says that we are working consistently below the W mass. The second says that the timescale for $SU(2)$ sector particle production is smaller than the timescale for the information about that process to be sent to the boundary.

1.1 Signature in chiral operators?

Now let's ask how to see this in QFT quantities such as

$$\langle Tr\Phi^n \rangle, \quad \langle T_{\mu\nu} \rangle \tag{10}$$

It is true that the particle production event cannot affect the *boundary value* of the field until time $t_{p,bdry}$. However, it affects the field itself immediately (up to suppression by powers of g_s that are required to source the bulk fields). One question is how to describe this in terms of operators like Φ^n and $T_{\mu\nu}$ before the time it takes the signal to reach the boundary.

Note that the AdS/CFT dictionary has a separate entry for VEVs, laid out for example by Klebanov/Witten, who describe how to write the moduli space in field theory variables using either VEVs of chiral operators or the eigenvalues they are made of.

The description in terms of VEVs of chiral operators requires keeping track of all $\langle Tr\Phi^n \rangle$ for all n up to N. One does not need to go to the boundary to describe this on the gravity side, in fact the higher Φ^n correspond to SUGRA fields which die very rapidly near the boundary (terms suppressed by $1/r^n$), and the D-brane which sources them sits at a constant radial position r_{probe}; it is never at the boundary.

It seems to me important that the AdS/CFT dictionary does not just concern correlation functions of local operators corresponding to sources inserted at the boundary: in the Lorentzian theory it separately requires specifying the state of the field theory, dual to normalizable fields, and in the Poincare version it requires specifying the VEVs (superselection parameters in the noncompact case).

The idea with $\int\ phi_{11}$ is to fix the gauge (say Landau gauge) and use the residual $SU(N)$ to rotate so that the VEV is in the 11 direction. One subtlety I could see with this is that the gauge fixing may break the SUSY, but there is a literature on how to treat this (it doesn't ruin the SUSY non-renormalization of the metric on moduli space). We could pursue this, but I don't get the feeling it is what is really bothering anyone.

It is true that the root mean square field is large at high momentum. But I don't see how this changes the fact that there is a VEV which is captured by $< \phi_{11} >$, or equivalently by $< Tr\Phi^n >$ (all n).

Now let's add in some dynamics – if we set the probes moving, for example scattering two probes at a spacetime point r, τ in the bulk. We want to identify when this happens in the dual gauge theory; the answer cannot depend on a time slicing choice in the bulk.

Let's use the hyperbolic slicing $-d\tau^2 + \tau^2 d\sigma^2$ for the metric the field theory lives on. The event happens at the time when the W boson has mass r/α' and the hyperbolic slice has curvature radius τ. These are invariant field theory quantities. When we compactify, we can also describe the latter as the time when the compact space the field theory lives on has size τ.

Another way to think about this is using RG ideas:

In an ordinary RG treatment, we can integrate down to the energy scale, Λ of the "W bosons" That is, we may consider a cut off version of the QFT, cut off at a scale

$$\Lambda_{cutoff} \gtrsim \phi \tag{11}$$

(This can even be done preserving all the SUSY, using T^6/Z_2 as in H. Verlinde's original paper on warped compactifications.) In the cutoff theory, we can calculate using a local Wilsonian effective action at a level of precision good up to corrections suppressed by powers of

$$Energy/\Lambda_{cutoff} \sim \sqrt{\dot{\phi}}/\phi \sim 1/\lambda^{1/4} \tag{12}$$

However, this energy cutoff does not correspond to the "holographic RG" prescription of bringing in the boundary from $r = \infty$ to

$$r_{cutoff} = \Lambda_{cutoff}\alpha' \tag{13}$$

since the energy scales corresponding to a give radial cutoff are different for objects of different proper energies in the bulk. In particular, although (13) does cut off the W bosons at the scale (11), but it cuts of the glueballs at a much lighter scale $\Lambda_{glue} \sim r_{cutoff}/R^2$.

Using, say, a holographic RG framework we can still calculate using the field theory operators Φ^n and T_{mn}, but if we did insist on integrating out the glueballs we would generate a somewhat nonlocal Wilsonian action. That's not a problem in principle, just a complication.

However I think the right strategy to take would be to recognize that the glueballs couple weakly to the $U(2)$ sector we are discussing, and not integrate them out but just keep track of this weak coupling. A possibly related idea would be to use the constructions that give the flat space S-matrix from AdS/CFT to probe this physics using chiral glueball operators that corresponds to bulk sugra fields.

The next step in this direction is to make the hyperbolic BHs out of it, but I want to make sure that the starting point is solid. I think the DBI action is the result of dynamics, and not something one could put in by hand to fool oneself that one has a spacetime, so it should break down if there is a big breakdown of EFT in old black holes. This is quite possible; my point is just that this is a question that can be set up in AdS/CFT.

2 Black hole timescales

Next, let us check the timescale for a brane at ϕ (in the U(2) sector) to hit the singularity of the corresponding hyperbolic black hole. This singularity is the slice $t_p = 0$ in the Poincaré coordinates given above. We need the particle production process to take place inside the horizon, let's say that we start it just inside; as discussed above it lasts a time of order $\Delta t_{p,part} \sim \dot{\phi}^{-1/2} \sim \lambda^{1/4}/\phi$ [1]. The horizon is the lightlike surface

$$t_{p,h} = -\frac{R^2}{r_{p,h}} \tag{14}$$

in Poincaré coordinates. The time it takes for our 2-brane subsystem to reach the singularity (the slice $t_p = 0$) is then of this order, i.e.

$$\Delta t_{p,sing} \lesssim \frac{\lambda^{1/2}}{\phi} \tag{15}$$

So we have a window

$$\Delta t_{p,W} \ll \Delta t_{p,prod} \ll \Delta t_{p,sing} \tag{16}$$

in which our particle production process completes parametrically before the singularity becomes important.

Another interesting scale is the KK scale on the compact hyperbolic space. This is of order

$$m_{KK} \sim \frac{1}{t_p} \sim \frac{\phi}{\sqrt{\lambda}} \tag{17}$$

So our particle production scale $\dot{\phi}^{1/2} < \phi/\lambda^{1/4}$ can be well above that scale. The dominant momentum modes produced in the particle production process are of order this scale $\dot{\phi}^{1/2}$, so they fit in our hyperboloid.

3 Supergravity modes inside the Horizon

The locus (14), corresponding to the horizon in the case of the compactified system (the black hole), comes up in analyzing the spectrum of supergravity modes ("glueballs"), in the following way. (This was a point Don Marolf made a while .)

If one looks for solutions of the massless wave equation (for example for scalars) in AdS with fixed Poincare momentum squared $k^2 = \omega^2 - \vec{k}^2$, these are peaked at

$$r_{k^2}^2 \sim R^4 k^2 \tag{18}$$

In the compactified system, the length and time scales available are limited, in the metric

$$\frac{r^2}{R^2}\left(-d\tau^2 + \tau^2 d\sigma^2\right) + \frac{R^2}{r^2} dr^2 \tag{19}$$

we have

$$k^{-1}, \omega^{-1} \leq \ell_\Sigma = \tau \tag{20}$$

That means that if we had $k^2 \sim \omega^2 \sim \vec{k}^2$, our spectrum of solutions would go down to a radial position

$$r^2_{1/\tau^2} \sim \frac{R^4}{\tau^2}, \tag{21}$$

a locus corresponding precisely to the horizon (14).

However, whether we compactify or not, we can obviously consider supergravity fields propagating in the region at lower radial position r than (21) even if their momentum is bigger than $1/\tau$. For one thing, we can consider $\omega^2 - \vec{k}^2 \ll \omega^2, \vec{k}^2$. We can also introduce a nonzero mass into our wave equation (for example, KK excitations on the S^5), which should presumably lead to solutions peaked at lower values of r. Finally, rather than considering a solution with fixed ω and $\vec{k}$, we can consider an initial wave packet which is localized at scales shorter than τ and also localized at a radial position well inside the horizon. These are all legitimate excitations of the system (normalizable solutions of the supergravity equations) which must have a description in the dual field theory. They must be described by an appropriate state of the dual QFT.

4 SUSY breaking scale

The Poincaré field theory lives on the Poincare space $-dt_p^2 + d\vec{x}^2$; including the redshift factors in the 5d metric, the DBI action is

$$-\int d^4x_p \left\{ (\phi^4/\lambda)\sqrt{1 - \lambda\dot{\phi}^2/\phi^4} - \phi^4/\lambda \right\} = \int d^4x_p \dot{\phi}^2 + \dots \tag{22}$$

When we work inside the patch corresponding to the shrinking Hyperboloid $-d\tilde{t}_p^2 + \tilde{t}_p^2 dH^2$, the SUSY breaking coming from the hyperboloid is therefore of order

$$M^{hyp}_{SUSY} \sim \frac{1}{\tilde{t}_p} \sim \frac{\phi}{\sqrt{\lambda}} \sim \frac{r_p}{\alpha'}\frac{\alpha'}{R^2} \sim \frac{r_p}{R}\frac{1}{R} \sim M^{EFT}_{SUSY} \tag{23}$$

The last expression expresses this SUSY breaking scale as a redshifted down proper energy scale of $1/R$, which is the curvature radius of the gravity side, hence my identification of it with the bulk EFT prediction for the SUSY breaking scale (in the energy conjugate to Poincare time). This is all going on inside the horizon but far before the usual singularity which occurs at $\tilde{t}_p = 0$.

It seems clear that in the presence of a firewall, the proper SUSY breaking energy would be much higher, perhaps cut off at the string scale. That would give

$$M^{FW}_{SUSY} \sim \frac{1}{\sqrt{\alpha'}}\frac{r_p}{R} \gg M^{EFT}_{SUSY} \tag{24}$$

where again the redshift factor is included.

In this model, the evaporation comes from brane decay, but like Hawking pairs these might seem to be treatable as separate independent events that don't wildly increase the SUSY breaking scale in themselves. But I really need to think more about what can be said precisely from the QFT using the SUSY algebra or something...

5 Fire Drill

A firewall would be a large, SUSY-breaking effect which we should be able to dectect or exclude in this model. In this section, we will only use the QFT. It lives on the space

$$-d\tau^2 + \tau^2 ds^s_{H_3/\Gamma} \tag{25}$$

with a Hubble rate

$$H(\tau) = -\frac{1}{\tau} \tag{26}$$

For some purposes it is useful to make use of the conformal transformation $\tau = -\ell e^{-\eta/\ell}$ to a static compact hyperbolic space times time.

5.1 Moduli space before compactification

The $\mathcal{N} = 4$ SYM has a moduli space of vacua R^6/S_N which is parameterized by zero mode eigenvalues of the scalar fields *** **express this using a concrete gauge fixing procedure** ***. The metric on moduli space is uncorrected, and the first two terms in the effective action for a zero mode eigenvalue

$$\mathcal{S}_\phi = \int d^4x \left\{ \dot{\phi}^2 + \frac{\lambda \dot{\phi}^4}{\phi^4} \right\} \tag{27}$$

are protected, and hence they remain valid at large 'tHooft coupling λ.

The effective action of the theory takes the form

$$\mathcal{S} = \mathcal{S}_{U(2)} + \sum_{n,m,p,\Delta} \frac{\partial^n \phi^m \mathcal{R}^p \mathcal{O}_\Delta C_{n,m,p,\Delta}(\lambda, N)}{m_W^{n+m+2p+\Delta-4}} + \mathcal{S}_{U(N-2)} \tag{28}$$

where $\mathcal{S}_{U(2)}$ includes $\mathcal{S}_\phi$ (27). For the fire drill we will be interested in a configuration of the full theory which includes nonzero VEVs and fluctuations of $N/2$ of the eigenvalues of the $U(N-2)$ sector.

5.2 Effect of compactification

The compactification introduces a BH on the gravity side, and at the same time leads to its decay via brane nucleation. In the η frame (convenient because of its time translation symmetry) one can estimate that they carry of order N^2 entropy out; as Juan pointed out this follows from the fact that they come out slowly ($\Delta\eta \sim e^{cN}$, implying a number of states from phase space of order $(e^{CN})^N \sim e^{CN^2}$). To have the putative firewall, we would need an old BH, meaning that $\sim N/2$ branes have come out.

5.2.1 Firewall prediction

From (14), we see that a firewall would arise at a time τ_h satisfying

$$\phi = -\frac{\sqrt{\lambda}}{\tau_h} \Rightarrow H(\tau_h) = \frac{\phi}{\sqrt{\lambda}} \tag{29}$$

That is, Hubble at the time of the putative firewall is tiny; equivalently, the hyperboloid is huge.

Note that these statements are being made on the QFT side, not on the gravity side where we would not be able to assume local QFT. Relatedly, this is expressed in terms of the ratio of the W mass and the Hubble scale, an invariant field theory quantity which does not depend on the time slicing in the bulk.

Therefore, we need to determine the effect of the large, slowly shrinking compactification on the moduli space approximation.

5.2.2 Effect on moduli space: topological sectors and dynamical moduli

Here we will consider two new effects introduced by the compactification: (i) the coordinates on the moduli space become dynamical rather than being superselection parameters, and (ii) there are additional topological sectors into the QFT, the Wilson lines and their canonically conjugate fluxes.

Point (ii) qualitatively changes the classical moduli space: instead of R^6/S_N it now contains[1]

$$(T^{b_1} \otimes R^6)/S_N \tag{30}$$

where T^{b_1} is the torus spanned by the commuting Wilson lines:

$$A_n = \sum_{I=1}^{b_1} a_n^{(I)} \omega^{(I)} \tag{31}$$

in terms of a basis of 1-forms ω; let us normalize these so that their periods are the length of the circle in the geometry $\oint_\gamma \omega = \ell_\gamma$ (similar to a convention on a circle $x \equiv x + \ell$ in which we write $A = a dx$). (This torus is formally the T-dual of the Jacobian torus in the case of a Riemann surface one dimension down.)

The size of the T^{b_1} is given by the periodicity of the Wilson lines, which determines their quantized energy levels. The R^6 directions produce a continuum of levels, with the scalar zero

[1] In fact there are classically of order N^2 new directions, but we suspect that only those that are protected by the symmetry of the T^{b_1} to survive quantum mechanically.

mode coordinates now dynamical. In this section we will analyze each of these effects on the low lying spectrum and the moduli space approximation.

We take a convention in which the action is schematically

$$\int d^4x\sqrt{g}\frac{1}{g_{YM}^2}\left(F^2+[(\partial+A)\Phi]^2+\dots\right)\to\int d\tau\hat{V}_\Sigma\frac{\tau^3}{g_{YM}^2}\left(\dot{A}^2+\dot{\phi}^2+\dots\right) \tag{32}$$

where in the last step we focus on the zero modes of ϕ and A, with the volume of our compact hyperbolic space given by $\tau^3\hat{V}_\Sigma$. To begin, let us analyze the energy levels and fluctuations of the zero mode eigenvalues ϕ and A by themselves. After that we will consider the effects of off-diagonal matrix entries.

The Wilson lines and their periodicity in this convention are

$$W_{\gamma_I}=P(e^{i\int_{\gamma_I}A})\Rightarrow a^{(I)}=a^{(I)}+\frac{2\pi}{\ell_{\gamma_I}} \tag{33}$$

(with no g_{YM}). In our case, the size ℓ_γ of each circle in the geometry is of order $\ell_\Sigma=\tau\hat{V}_\Sigma^{1/3}$. The circle is small for large ℓ_Σ, but the effective mass of the Wilson line variables A is large, $m_A\sim\ell_\Sigma^3/g_{YM}^2$ (32). As a result, the quantized momentum levels from (32)(33) scale like

$$\frac{p_a^2}{2m_A}\sim\frac{\frac{n_a^2}{1/\ell_\Sigma^2}}{\ell_\Sigma^3/g_{YM}^2}=\frac{g_{YM}^2n_a^2}{\ell_\Sigma} \tag{34}$$

for integer n_a. This applies to each of the N commuting Wilson lines, in each of the b_1 directions.

Next let us consider the dynamics of the zero mode eigenvalue of ϕ by itself. The R^6 directions in (30) produce a continuum of levels, with the scalar zero mode coordinates now dynamical. Although dynamical, their fluctuations are controlled by the large size of the H^3/Γ the and small rate of its Hubble shrinking. Here we will analyze this for a single zero mode eigenvalue by itself. The uncertainty principle gives

$$\Delta\phi\Delta\dot{\phi}\geq\frac{g_{YM}^2}{|\tau|^3} \tag{35}$$

This means that for a minimal uncertainty wavepacket, we have a fluctuation in ϕ of order

$$\Delta\phi\sim\frac{g_{YM}}{|\tau|}\ll\frac{\sqrt{\lambda}}{\tau} \tag{36}$$

The last inequality means that at the time of the putative firewall, the fluctuation in the zero mode eigenvalue is negligible compared to its value, $\Delta\phi/\phi\ll 1$. So although the problem has become quantum mechanical as a result of the compactification, the moduli space approximation remains parametrically well controlled against this specific effect. For our probe, we would consider a wavepacket centered at $\phi_h\sim-\sqrt{\lambda}/\tau_h$.

Now let us consider the effect of the off-diagonal modes and interactions on the spectrum and moduli space. Before we compactified and got the additional directions $a^{(I)}$, we had an exact, uncorrected moduli space geometry R^6/S_N. (Moduli trapping effects strongly affect the dynamics already in that case, but the geometry of the moduli space is robust.) However, including the new T^{b_1} directions, we now have a *non-supersymmetric* matrix quantum mechanics at low energies. Let us estimate the effect of this out on the R^6 directions parameterized by ϕ, integrating out loops of "W bosons" in the quantum mechanics theory. These now have mass squared of the schematic form $\phi^2 + a^2$, with the contribution depending on a breaking the supersymmetry (i.e. the mass shift $\propto a$ is not the same for bosons and Fermions). As a result, whereas in D0-brane quantum mechanics one has a cancellation in the quantum effective potential and $\dot{\phi}^2$ term, here these will scale like

$$\int d\tau \left\{ N\frac{a^2}{\phi} + \frac{\ell_\Sigma^3}{g_{YM}^2}\dot{\phi}^2(1 + \frac{g_{YM}^2 N a^2}{\ell_\Sigma^3 \phi^5}) \right\} \tag{37}$$

We can write this as (restoring $\ell_\Sigma \sim \tau$ at this point)

$$\int d\tau \frac{\tau^3}{g_{YM}^2} \left\{ \lambda \frac{a^2}{\tau^3 \phi} + \dot{\phi}^2(1 + \frac{g_{YM}^2 N a^2}{\tau^3 \phi^5}) \right\} \tag{38}$$

Since $a \leq 1/\ell_\Sigma \sim 1/\tau$, we see that the correction to the moduli space metric is bounded as follows

$$\frac{g_{YM}^2 N a^2}{\ell_\Sigma^3 \phi^5} \leq \frac{\lambda}{\ell_\Sigma^5 \phi^5} \to \lambda^{-3/2} \tag{39}$$

with the last arrow applying for our probe on the horizon, for which $\phi \ell_\Sigma \sim \sqrt{\lambda}$.

Including the induced potential term we have the equation of motion[2]

$$\ddot{\phi} = -\frac{3}{\tau}\dot{\phi} \pm \frac{\lambda}{\tau^5 \phi^2} \tag{40}$$

This includes anti-Hubble-friction ($\tau < 0$ on the shrinking cone). The $\pm$ here is because I have not determined which direction the SUSY breaking goes. For our probe $\phi = \phi_{pr}$ we are interested in times earlier than and up to around the horizon time (29), which means

$$\phi \tau \geq \sqrt{\lambda} \tag{41}$$

Plugging this into the equation of motion (40), we see that the potential term is negligible for the early history of the system, and then becomes marginally competitive with the other terms at the horizon, where the potential term goes like $1/\tau^3$. Although it becomes of the same order as the other terms here, this is far from a firewall, since all the terms lead to mild evolution in the region of field space dual to the horizon crossing.

A final remark: with the new Wilson line degrees of freedom, there exist "long string" configurations where the N Wilson lines spread out into an array on the T^{b_1}. It would be interesting to understand if these are important for the entropy (including the entanglement entropy that develops with radiated eigenvalues), as in previous examples of microscopic black holes.

[2] Note that $\ddot{\phi}$ is different from the proper acceleration on the gravity side; for our probe we want to work at small velocity so that DBI is not necessary.

5.2.3 Electric flux condensation and energy gap: firewall/fuzzball candidate

In the example [2], one compactifies the N=4 SYM on a Scherk-Schwarz circle. On the gravity side, the system caps off into the AdS bubble solution of Horowitz and Myers. In the setup [2], this happens dynamically, in a process that starts with a winding tachyon condensate which caps off the geometry first at much smaller r than it ends up. That is, it caps off when the formation shell gets below a radial position r_{tach} for which the compact geometry has string scale size:

$$\ell_\Sigma \frac{r_{tach}}{R} \sim \sqrt{\alpha'} \tag{42}$$

But eventually the system ends up with the cap up at a higher radial scale corresponding to the AdS bubble solution:

$$\ell_\Sigma \frac{r_{confinement}}{R} \sim R \tag{43}$$

Note that this latter scale leads to supergravity "glueball" modes gapped at a scale of order $1/\ell_\Sigma$.

Now in this example, from the gravity side we see that the retraction process cannot happen faster than the speed of light, $\int dr/r^2 = \int dt_p/R^2$, i.e.

$$\Delta t_p \geq \frac{R^2}{r_{tach}} = \ell_\Sigma \lambda^{1/4} \tag{44}$$

It seems that in this case, we can also estimate this timescale from the field theory side as follows. Before the retraction occurs, we have solutions to the massless supergravity wave equation which are localized in the region between r_{tach} and r_{conf}. These have a characteristic size in the spatial directions $x_{1,2}$ on which the field theory lives (the directions transverse to the circle along say x_3) which is given by

$$k^{-1} \sim \frac{R^2}{r} \tag{45}$$

as in (18). From this we immediately see that in the field theory alone, by field theory causality that this mode takes at least the time (44) to shrink from its initial size of $R^2/r_{tachyon}$ to the much smaller size R^2/r_{conf}.

In our case, we may have a similar confinement/capping off effect, since we are compactifying all directions on a SUSY breaking space. (For Riemann surfaces, even if one takes a periodic spin structure there are still Scherk-Schwarz circles in the geometry, and also we could consider a more general spin structure.) Now we note:

• The radius (43) is precisely the horizon locus in our case, so this is an interesting candidate for a fireball.

Now the naive gravity side would say there is not enough time for this process to affect the region near the horizon; the tachyon condensate is limited to a region close to the BH singularity. However, we cannot conclude anything directly from the gravity side after compactification; we must use the QFT side.

So next, let us ask whether there is an analogue of the QFT argument we made in the previous example, where we used field theory causality to bound the time it takes to complete the shrinking of modes.

Interestingly, in the compact geometry, if the geometry is isotropic the modes of fixed $\omega^2 - \vec{k}^2$ never grow larger in size than they are at the horizon, as discussed above in §3. So that QFT argument does not directly apply to this case. It may be that a more subtle version of the argument involving long string configurations may come in.

However, if it is anisotropic – in fact there are hyperbolic spaces which have finite volume but noncompact directions – then we *do* have long wavelength modes inside the horizon. To these we may be able to apply the above causality argument.

As a final remark, we can also of course consider the intermediate case of Scherk-Schwarz compactification on a T^3 (rather than just an S^1). There, there is no black hole and the gravity side arguments should hold, but because it is fully compactified in the spatial directions, this case has the same limitation on the field theory argument as in our case.

So to sum up: we need to determine whether the system gaps up (due to electric flux condensation), removing the region $r < r_{conf}$, by the time $\tau_{N/2}$ at which $N/2$ of the eigenvalues have tunneled out. In the next section we will lay out the timeline of events in our system, finding that the formation occurred at a double exponentially long time in the past; this corresonds to the lightest windings going tachyonic also at a doubly exponential time in the past.

5.2.4 Effect of $N/2$ emitted eigenvalues on the probe sector

We are interested in probing our system after it has undergone $N/2$ eigenvalue emissions, after which at low energies we have symmetry breaking $U(N+1) \to U(N/2) \times U(1)^{N/2} \times U(1)_{probe}$. (Here we write $U(N+1)$ to describe the probe as an additional system we are dropping into the BH which we earlier made from N eigenvalues.) In this section, we analyze their effect on the probe dynamics.

First, let us record some of the basic timescales and solutions that will come into this. In the static frame with time η,

$$\tau = -\ell e^{-\eta/\ell} \tag{46}$$

we should have a timescale between emissions of order

$$\Delta\eta = \eta_2 - \eta_1 \sim \ell e^{BN} \tag{47}$$

where B is a constant of order 1. Back in our original coordinates, this means

$$\tau_1 \sim \tau_2 e^{e^N} \tag{48}$$

The time τ_1 of the previous emission is double-exponentially far in the past, as is the formation time. We may for example set up the system so that the last of the $N/2$ eigenvalues comes out at some time $\tau_{N/2}$ which is independent of N. Then earliest brane came out at an earlier time τ of order $\sim \tau_{N/2} e^{(N/2)e^{BN}}$, double exponentially far back as a function of N, and we formed the BH at a time $\tau_{form} \sim \tau_{N/2} e^{(N/2+1)e^{BN}}$.

If we assume that the branes nucleate near the horizon, we have outgoing solutions for the emitted eigenvalues of the form (using just the quadratic action for the moment, which should be valid at late times)

$$\phi \sim \sqrt{\lambda}\frac{\tau_{nucl}}{\tau^2}, \qquad \dot{\phi} \sim -2\sqrt{\lambda}\frac{\tau_{nucl}}{\tau^3} \tag{49}$$

(This satisfies (14) at $\tau = \tau_{nucl}$). This solution is consistent at late times $\tau \ll \tau_{nucl}$, since

$$\frac{\lambda\dot{\phi}^2}{\phi^4} \sim \frac{4\tau^2}{\tau_{nucl}^2} \ll 1 \tag{50}$$

so the DBI corrections do not matter at late times. The other solutions to the equations of motion in Poincare frame are simply constant, and our probe satisfies this to leading approximation

$$\phi_{probe}(\tau) = \phi_{pr} = \frac{\sqrt{\lambda}}{\tau_h} \tag{51}$$

where as in (14), τ_h is the field theory time corresponding to the D-probe crossing the horizon.

Let us check whether our probe sector and any of the $N/2$ other $U(1)$ sectors interact appreciably during the process. In particular, we would like to check whether moduli trapping occurs, since a priori that might obstruct our probe or affect the entanglements in the system. In that process, the number density of off-diagonal modes goes like

$$n_W \sim \int d^3\vec{k}|\beta_{\vec{k}}|^2 \tag{52}$$

where the Bogoliubov coefficient is

$$\beta_{\vec{k}} \sim e^{-(\vec{k}^2+\Delta\phi_{min}^2)/|\Delta\dot{\phi}|} \tag{53}$$

where $\Delta\phi_{min} \sim \phi/N^{1/5}$ is the minimal distance along the moduli space that the two eigenvalues reach, using that the emitted ones are uniformly distributed in the angular directions of the R^6.

Using our solution (49), we have

$$e^{-\Delta\phi_{min}^2/|\Delta\dot{\phi}|} = e^{-\sqrt{\lambda}\tau_{nucl}/(N^{2/5}\tau_{trap})} \tag{54}$$

where τ_{trap} is the time at which the probe comes closest to the outgoing eigenvalue. One way to express this given our timeline of events is as follows. From (49)(51) we can equate the values of ϕ for the probe and the outgoing eigenvalue to obtain

$$\sqrt{\lambda}\frac{\tau_{nucl}}{\tau_{trap}^2} = \frac{\sqrt{\lambda}}{\tau_h} \Rightarrow \tau_{trap} = \sqrt{\tau_{nucl}\tau_h} \tag{55}$$

This gives for (54)

$$e^{-\frac{\sqrt{\lambda}}{N^{2/5}}(\tau_{nucl}/\tau_h)^{1/2}} \tag{56}$$

Now, we can send the probe in so that the time τ_h is of order the time $\tau_{N/2}$ we described above, so as to probe the system at about the Page time. This means $\tau_{nucl}/\tau_h \sim e^{ne^{BN}}$ for some $n \leq N/2$. As a result, the Bogoliubov coefficient (53) is *triply* exponentially suppressed. That means that the number density of off-diagonal modes is much smaller than the inverse volume of the compact hyperboloid at the time τ_{trap}, since the latter is a measely double exponential.

5.2.5 Full theory

Of course our quantum mechanical eigenvalue ϕ and Wilson lines a are a small part of the full theory (28). So we should also check whether the compactifcation has a large effect on the rest of the theory including its interaction with the zero mode of ϕ. We want to check that correlators $\langle \mathcal{O}(x_1)\dots\mathcal{O}(x_n)\rangle$ get corrections of order $H|x_{ij}|$ upon compactification, that is that the theory reduces to the original QFT on scales shorter than H^{-1}. In particular, we wish to understand if this crossover scale in the compactified theory is instead strongly corrected at large λ, as would be required by a Firewall.

On general grounds, causality will require waiting a timescale of order $|\tau|$ between the start and end of any process in order to see effects of the compactification. A putative firewall would flare up on a much shorter timescale.

In general, strong λ dependence does not seem to arise: consider a CFT on a circle for example. λ affects the spectrum of dimensions (giving a big hierarchy) in the theory, and also affects the OPE coefficients (giving corrections dual to α' effects, which are suppressed at large λ). The large hierarchy of dimensions suppresses propagation around compact cycles in the geometry: we expect suppression of the form $\sim e^{-\Delta\tau/\ell}$ from this. On a torus, we obtain a trace over states

$$\sum_n \langle n|\mathcal{O}\mathcal{O}e^{-\Delta\tau/\ell}|n\rangle \tag{57}$$

which can be traded for a sum over four-point functions, in turn determined by OPE coefficients and operator dimensions, neither of which seems to give a large effect at large λ.

In general, the firewall would imply a large effect at the time τ_h given above in (29), where $\lambda H(\tau_h)^2 \sim \phi^2$. To play with this a little more, ϕ^2 is physically the (near-)BPS W boson mass squared. A large effect would arise if somehow there were a correction like $-\lambda H^2 W^2$, cancelling the W boson mass squared. From its form it would arise from the a combination of the compactification, which gives $H \sim 1/\tau > 0$, and would have to come with a large λ enhancement (the sort of thing that sometimes arises via loops with N species). This large enhancement does not seem to happen, either from the CFT point of view or using the gravity side (say for a young BH). But we should think more to nail this all down. In particular, the role of the already-nucleated $N/2$ eigenvalues after the Page time needs to be considered carefully.

6 More on decay quantum mechanics and entropics

If there's no FW then something else has to give, either one of the postulates or the assumption that the measurement involved was operationally possible. It is worth analyzing the decay process in more detail, to understand the manifestation of the paradox in this system. (But note that we may be able to exclude a FW without solving the problem of what it is that gives.)

Scattered comments:

1) As mentioned above, the compactification introduces a BH on the gravity side, and at the same time leads to its decay via brane nucleation. In the η frame (convenient because of its time translation symmetry) one can estimate that they carry of order N^2 entropy out; as Juan pointed out this follows from the fact that they come out slowly ($\Delta\eta \sim e^{cN}$, implying a number of states from phase space of order $(e^{CN})^N \sim e^{CN^2}$). To have the putative firewall, we would need an old BH, meaning that $\sim N/2$ branes have come out.

2) From an old email: I looked into the spreading of the wavefunction in the cylinder frame, translating eqns (14)-(18) of hep-th/0312163 trivially into our language.

The result is, for $X = R\phi$:

$$\psi \propto Exp[-2(X - X_0(\eta))/(\Delta X(\eta)^2)]$$

where $X_0(\eta)$ is our classical solution $\propto cosh\eta$ and the spread is given by

$$(\Delta X)^2(\eta) = (\Delta X)_0^2 + sinh^2\eta((\Delta X)_0^2 + 4/\Delta X)_0^2)$$

From this we see that if we minimize the spread at infinity by taking $(\Delta X)_0$ of order 1, (the spread at infinity goes to infinity, but the ratio $X/(\Delta X)$ does not diverge), then we get the spread $\Delta\phi \sim 1/R$ at the origin. If we take the spread at the origin to be smaller, then we get a larger spread away from $\eta = 0$.

We also saw immediately from Albion's formula the usual result that if we insisted on starting with a delta function wavepacket, it would spread out immediately as usual.

If I'm not missing any important factors of N, this is saying that $\Delta X/X$ becomes larger than one for $\phi < 1/R$. But before we found that DBI corrections became important earlier, already for $\phi < R/\alpha'$.

3) Maybe we could mine the black hole to extract the entropy faster, perhaps using a baryon, another kind of D-probe which comes with strings that stretch out to the boundary.

7 some questions

Here is a (rough) summary of the questions we raised today.

A) Diagnostics of behind the horizon physics: (any one of them will produce a large firewall signal)

a) D-brane probe:

Causality problem for local gauge invariant operators. Does using $tr\Phi^k$ for all k less than N, lessen the causality problem?

For gauge fixed ops what gauge to work in?

b) Effective theory on a fixed r surface:

Does such a theory, obtained e.g., from the holographic RG, make sense as a field theoretic object?

In a and b above, what is the most robust way to look for a firewall signal? Susy breaking, Conformal Inv breaking?

B) Field Theory on collapsing cone (This seems like a good way to directly expose behind the horizon physics)

a) What effects does modding out the field theory space (to get a compact hyperbolic space) cause?

Winding modes, energetics,

Torus as a warmup. Understand AdS/CFT on T^3. What are light states in field theory that are dual to light winding strings in bulk?

(T duality in bulk for (spatial S^1) discussed v. briefly in hep-th/0508077, Aharony et al. p 42)

Scherk-Schwarz condensation–blow up from string to AdS scale

b) brane decay process

c) analog of finite temperature viewpoint in collapsing cone coords.

8 Random remarks

I noticed that for a Gaussian wavefunction centered at the origin,

$$\Psi[\phi] \sim e^{-\phi^2/\Delta\phi} \tag{58}$$

if we evaluate this anywhere on the horizon (14) it becomes something simple:

$$e^{-\phi^2/\Delta\phi}|_{horizon} \sim e^{-\lambda/g_{YM}^2} = e^{-N} \tag{59}$$

Anyway, the wavefunction of the full system is more complicated, at least near the origin.

Bibliography

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